In Ref. 2, Libby did not write down Eq. (4), but instead asserted "it is readily clear that for $x > L$, $t \equiv t_c$" and proceeded to apply $t(L) = t_c$ as a boundary condition on Eq. (3). At this point we stress that Eq. (4) is a necessary part of the mathematical description of the problem posed by Libby in Ref. 2; inclusion of gas conduction requires that the solutions in the two regions, $0 \leq x \leq L$ and $L \leq x < \infty$, be correctly matched. The solution of Eq. (4) is

$$t = C_1 \exp[(\rho v c_p/\lambda_g)x] + C_2 \quad L \leq x < \infty \tag{5}$$

For suction v is positive and Eq. (5) shows that t grows exponentially with x. Thus it is not possible to impose a Dirichlet type boundary condition as $x \to \infty$, which is what Libby essentially did by setting $t \equiv t_c$. Instead the correct boundary conditions on Eq. (4) are

$$x = L: \qquad t(L^+) = t(L^-) = t(L)$$

$$x \to \infty: \qquad t' = 0 \text{ (no internal refrigeration)}$$

and the correct solution of Eq. (4) is therefore

$$t \equiv t(L) \tag{6}$$

where $t(L)$ is as yet still to be determined. The additional four boundary conditions required for the sixth order mathematical system are

$$x = 0: \qquad -\lambda_s T' - \lambda_g t' = q_{\text{conv}} - q_{\text{rad}} \tag{7}$$

$$T = t \tag{8}$$

$$x = \text{L}: \qquad -\lambda_s T' = 0 \tag{9}$$

$$-\lambda_g t(L^-) = 0 \text{ [since } t'(L^+) = 0] \tag{10}$$

Solution of Eqs. (2) and (3) subject to these boundary conditions yields the trivial solution

$$T = t = \text{const} = T_w \tag{11}$$

and T_w is determined by Eq. (7) which degenerates to

$$0 = q_{\text{conv}} - q_{\text{rad}} \tag{12}$$

Thus we have shown that $t_c = t_w$ (or $h_c = h_w$) is the correct solution of the suction problem. We note in passing that in Ref. 2, Libby went further astray by not recognizing that Eq. (9) must hold, i.e., the physical requirement that the heat flux in the solid at $x = L$ must be zero, unless for example, cooling coils are brazed onto the backface of the wall.

Returning to our criticism of Ref. 1, where solutions were obtained for prescribed $h_{c,2} < h_w$, it is clear that such systems will require internal refrigeration. This additional thermodynamic burden is contrary to the authors' intent. Furthermore, an adiabatic system will require the suction region value of h_w to be lower than its prescribed value in the injection region. Thus the convenient simplifications which result from the assumption of position independent h_w and q_{rad}, and which were exploited by Libby and Hendricks, would not apply to an adiabatic system. Finally, although ingested air is a possible source of transpiration coolant, it has yet to be shown that the scheme considered in Ref. 1 has adequate potential for the proposed applications.

References

[1] Libby, P. A. and Hendricks, P., "Analysis of an Active Thermal Protection System for High Altitude Flight," *AIAA Journal*, Vol. 8, No. 9, Sept. 1970, pp. 1671–1678.

[2] Libby, P. A., "Reply by Author to A. F. Mills and R. B. Landis," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 379–380.

[3] Libby, P. A., "Temperature Distributions in Porous Surfaces with Either Suction or Injection," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1206–1208.

Reply by Author to A. F. Mills

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For the reader who appreciates clarity and lacks the time to assess for himself a technical controversy, it is regrettable that Professor Mills did not avail himself of the opportunity we afforded him to comment prior to submittal on our rejoinder to his first criticism. If he had done so, we could perhaps have developed a single response and avoided his present comment.

Professor Mills' objection can be succinctly stated; he requires that both conduction terms, the one in the solid, $\lambda_s T'(L^-)$, and the one in the gas $\lambda_g t'(L^-)$, be individually zero whereas we require only that their algebraic sum be zero. Despite Professor Mills' present argument we see no physical or mathematical reason to impose this extra requirement and believe our analysis to be correct. Thus the reader must expend some time if he wants clarification of this matter.

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